VALUING FLOATING RATE BONDS (FRBS)

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Valuing Floating Rate Bonds (FRBs)

1. The principal features of floating rate bonds can be summarised simply: these are bonds having a fixed maturity, sometimes with call/put options, but with the coupon refixed periodically with reference to a well-defined benchmark (the "reference rate"). The refixation could be daily (overnight MIBOR-linked bonds), or at 3/6/12 month intervals. In international markets, the most common benchmark is LIBOR. In India, apart from MIBOR, T-bill yields, other government bond yields, CP yields, MIFOR, etc, are also used. In principle, the benchmark reference rate and refixation periods may not have the same maturity – for example, the benchmark could be, say, the ruling yield on 360 day T-bill, coupon to be refixed every six months. In general, the floating rate will be at a spread over the benchmark, the spread reflecting the credit risk of the issuer, the maturity of the bond and its liquidity, demand for that type of instrument in the market etc. In India, both GoI and corporate issuers have issued FRBs.

   In some ways, the valuation of floating rate bonds is conceptually and mathematically more complicated than the fixed rate variety, particularly when the desired spread, or premium, over the reference rate differs from the contracted spread.

2. We will consider a simple case first: a floating rate bond with coupon equal to yield on 180 day T-bill, refixed every 6 months, and only one refixation left, 90 days from now. Again, for the sake of implicitly, we consider a 360 day year.

3. Valuation considering only the current floating rate

a. Assume the current coupon to be 5 % p.a., fixed on 27/7, refixation on 27/1

b. Valuation being done on 27/10 when the T-bill yield *for the balance period of 3 months* has moved to say 6% p.a.

c. The price reverts to par on the coupon refixation day.

d. Therefore, the full price of the bond on 27/10 will be (1.025/1.0150). The numerator includes the coupon amount due on 27/1 and the par price of the bond on that day. The denominator, or the discount rate, is 6% p.a. for the balance period of 90 days as valuation is being done on 27/10. The full price on that day works out to 1.009852.

4. Is it not necessary to consider the future coupon(s), for calculating today's price? While the coupon is not known, it could be locked into by entering into a forward rate agreement (3*9 in the cited case). Moreover, the money market/zero coupon rate for the coupon inflow date can be used for discounting the inflow. It can be shown, however, that, under the assumed conditions, i.e. zero premium over the reference rate and reversion to par value on coupon refixation day, there is no need to bring in the future coupon(s) or the forward rates, for

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calculating the price: the two methods give the same answer as the following calculation illustrates:

Valuation considering the ruling 3*9 forward rate, when 3 month T-bill yields 6% p.a. and the 270 day T-bill yields 8% p.a.

a. The two zero coupon rates are

   6% p.a. 90 days (i.e. 0.015)

   8% p.a. 270 days (i.e. 0.06)

b. The 3*9 months forward rate is ((1.06/1.015) - 1)*2 = 8.8670% p.a., and this is the coupon implied by the term structure for the period 27/1 to 27/7, i.e. the maturity of the bond

c. Therefore, using the forward rate as the coupon refixation rate, the full price on 27/10 is

   (0.025/1.015)+(1.044335/1.06)

   (discounting inflows by the ruling money market/zero coupon rates, noting that principal now comes only at the end of 9 months)

   = 0.024631 + 0.985222 = 1.009853

   which is the same as calculated by using only the current floating rate (ignoring the rounding up difference in the 6th decimal).

   The reason is simple: look at 3(d). The numerator is 1.025 as against 0.025 in the first term of 4(c) (the denominator is the same). However, the PV of the difference in the numerator, namely 1 discounted for 3 months, i.e. 1/1.015 is 0.985222, which is the same as the second term in 4(c).

5. As the following spreadsheet incorporating a number refixations illustrates, using the existing coupon alone, and using existing forward interest rates for future coupons, give the same answer.

   (Note that, in the spreadsheet, *ZCYs for the*

1. simple yields for T bills						
2. ZCYs for maturities beyond 6 months – expressed on bond equivalent basis.						
contracted spread =				**desrd sprd**	**0.0000**	
	0.25	**0.75**	**1.25**	**1.75**	**2.25**	**2.75**
	t	**.5+t**	**1+t**	**1.5+t**	**2+t**	**2.5+t**
ZCYs	4.25%	4.50%	4.65%	4.88%	5%	5.17%
Simple ZC rates	0.0106	0.0338	0.0591	0.0880	0.1175	0.1507
Forward rates	–	0.0229	0.0246	0.0273	0.0271	0.0297
Coupon incl spread	0.0250	0.0229	0.0246	0.0273	0.0271	0.0297
Simple Disc incl desrd sprd	0.0106	0.0338	0.0591	0.0880	0.1175	0.1507
FP based on 1st cpn alone	**1.014224**					
FP based on all cpns	0.024737	0.022135	0.023192	0.0250733	0.024252	0.894834
						1.014224
				Difference %	**0.0**	

exact maturity dates of the FRB coupons need to be used by interpolating standard maturity ZCYs. Again, in the following row, "simple ZC rates" have been calculated by converting periodically compounding ZCYs in the earlier row, to simple interest rates, for easy calculation of the forward and discount rates.)

In short, where there is zero spread over the reference rate, in terms of the coupon and current market conditions, calculating the price on the basis of the current coupon alone is sufficient.

6. Will the methodology be different where there is a contracted spread over the reference rate? We need to consider two separate situations:

a. There is a contracted spread over the reference rate, and it has not changed at the time of valuation; and

b. The currently desired spread over the reference rate is different from that originally contracted.

7. Considering the first situation, we can look at the problem of valuation from two different perspectives:

a. Since the contracted and desired spreads are identical, the price will revert to par on the date of refixation of the coupon. Therefore, it is sufficient if only the current coupon and the par value are considered, and discounted for the balance number of days till refixation of the coupon. The *discounting rate will need to be x% p.a. above the reference rate*.

b. Another way of arriving at the price is to adjust

1. simple yields for T bills						
2. ZCYs for maturities beyond 6 months – expressed on bond equivalent basis.						
contracted spread =		0.0100		**desrd sprd**	**0.0100**	
	0.25	**0.75**	**1.25**	**1.75**	**2.25**	**2.75**
	t	**.5+t**	**1+t**	**1.5+t**	**2+t**	**2.5+t**
ZCYs	4.25%	4.50%	4.65%	4.88%	5%	5.17%
Simple ZC rates	0.0106	0.0338	0.0591	0.0880	0.1175	0.1507
Forward rates	–	0.0229	0.0246	0.0273	0.0271	0.0297
Coupon incl spread	0.0250	0.0279	0.0296	0.0323	0.0321	0.0347
Simple Disc incl desrd sprd	0.0131	0.0413	0.0721	0.1067	0.1423	0.1819
FP based on 1st cpn alone	**1.011721**					
FP based on all cpns	0.024676	0.026777	0.027575	0.0291674	0.028104	0.875452
						1.011752
				Difference %	**0.00**	
		0.0278	0.0297	0.0323	0.0321	0.0347
		0.0328	**0.0347**	**0.0373**	**0.0371**	**0.0397**

the coupons calculated from the implied forward rates, and the ZCY rates, by x% p.a. for cash flow and discounting rate purposes, in the above spreadsheet.

The results are as under:

It will be noticed that there is a minor difference in the prices under the two methods. The reason is that the discounting of cash flows has been done at a rate different from the one used for calculating the forward rates, because of the spread over the reference rate. The first method, i.e. using only the current coupon, is more correct because, when the contracted and desired spreads are equal, the price reverts to par on coupon refixation.

8. We now look at the remaining situation where the contracted spread is x% over benchmark but the desired spread at the time of valuation is y%. In this case, the assumption of par value at the date of coupon refixation is not valid, since each coupon will have to yield a return different from the contracted coupon. The price therefore will differ from par.

9. The steps for valuing the bond in these circumstances would be as under:

a. Calculate the future coupons, inclusive of the contracted spread, based on the forward rates implied by the term structure.

b. Discount these at the zero coupon rates applicable for the date of receipt of the inflow, plus the desired spread which is y% p.a.

c. The full value then is the sum of the present values of all the coupons and the face value.

10. A spreadsheet with x = 1% p.a. and y = 2% p.a. is appended.

1. simple yields for T bills						
2. ZCYs for maturities beyond 6 months– expressed on bond equivalent basis.						
	contracted spread =	0.0100		**desrd sprd**	**0.0200**	
	0.25	**0.75**	**1.25**	**1.75**	**2.25**	**2.75**
	t	**.5+t**	**1+t**	**1.5+t**	**2+t**	**2.5+t**
ZCYs	4.25%	4.50%	4.65%	4.88%	5%	5.17%
Simple ZCrates	0.0106	0.0338	0.0591	0.0880	0.1175	0.1507
Forward rates	–	0.0229	0.0246	0.0273	0.0271	0.0297
Coupon incl spread	0.0250	0.0279	0.0296	0.0323	0.0321	0.0347
Simple Disc incl desrd sprd	0.0156	0.0488	0.0843	0.1235	0.1633	0.2069
FP based on 1st cpn alone	**1.009231**					
FP based on all cpns	0.024615	0.026585	0.027264	0.0287328	0.027596	0.857272
						0.992065
			Difference %	**-1.7**		

11. It will be noticed that, in this case, the price based only on the current coupon differs significantly from that using all the inflows based on the implied forward rates. Arguably, the latter is the more correct full price as the assumption of price reverting to par on coupon refixation, which is implicit in the former, is invalid in present case.

12. What has been described in paragraphs 8, 9, 10 and 11 above is a rigorous system based on the theory of bond pricing. In practice, however, it is not used by market practitioners. Apart from its complexity (which can be easily taken care of by specialised software), the reason for its not being used in the market is the need for interpolation of zero coupon curves for the coupon inflow dates. There is no uniformity in market practice in respect of the interpolation techniques to be used for drawing yield curves. There are various techniques from simple linear, logarithmic, least square polynomials, cubic splines, regression models, etc. All these lead to different interpolated numbers, often differing from each other significantly.

13. An alternate approach: Calculating issue price

Eugene Fama, well known for his pioneering work in formulating the efficient market theory, developed an equation for the expected price of an asset at a future date based on its current price and the return expected by the investor from now until the future date; the investor will not make the investment at the current price unless his expectations of the future price satisfy his desired rate of return from the investment.

This approach to future prices can be used to develop a model for the current, or issue, price of a floating rate bond. In fact, as historically one of the largest issuers of floating rate bonds, the Italian government's pricing of such bonds was based on the model. However, this pricing model requires two major assumptions to be made in terms of the future coupons:

- the current (i.e. at the time of valuation) value of the benchmark interest rate will continue through the life of the bond; and
- the market expectations of margin over the benchmark would also remain unchanged.

Fama's model can be used because the final value of the principal amount of the bond is known; by definition, it is the face value. From this, using the cash flows as given by the current value of the benchmark plus the contracted spread over benchmark, discounted at the benchmark plus the desired spread, gives the present value. It should be noted however that if the valuation is being done on a date other than the coupon refixation date, the first coupon, which is known, would need to be discounted at the current value of the benchmark plus the desired spread, that too for the fraction for the interest period which is still to run before refixation of the coupon. The mathematical derivation of the price formula is described below:

Notation used

P_0 - price on issue date;

P_1 - price on next coupon refixation date;

r - ruling reference rate

x - contracted spread over reference rate; and

y - desired spread over reference rate

n - number of periods to maturity

(r, x, y in fraction; face value 1)

Therefore,

r + y = desired yield

$1 + r + y = \frac{P_1 + r + x}{P_0}$

Or $P_1 - P_0 = P_0(r + y) - r - x$

But, on the assumptions of an unchanged reference rate through the life of the bond, and the desired spread,

$$P_0 = \sum_{i=1}^{n} \frac{r + x}{(1 + r + y)^i} + \frac{1}{(1 + r + y)^n}$$

and

$$P_1 = \sum_{i=1}^{n-1} \frac{r + x}{(1 + r + y)^i} + \frac{1}{(1 + r + y)^{n-1}}$$

$$P_1 - P_0 = (y - x) * (1 + r + y)^{-n}$$

Similarly,

$P_2 - P_1 = (y-x) * (1+r+y)^{n-1}$

Where P_2 is price at the end of two periods. Since P_n, i.e. face value, is 1, P_0, the issue price can be calculated as

14. The generally accepted market price, including in the global FRN market, uses the alternate approach desired in paragraph 13 above. For secondary market pricing in between coupon dates, the methodology may be summarised as under:

a. the current coupon is discounted at the ruling benchmark plus the desired spread , for the balance of the interest period;

b. the benchmark rate available on the date of valuation plus the contracted spread, are assumed to be the remaining coupons through the balance life of the floating rate bond, up to its maturity;

c. inflows subsequent to the first, are discounted at the assumed coupon plus/minus the difference between contracted and desired spreads;

d. the sum of the present values gives the full price of the floating rate bond on the date of valuation.

This methodology gives the same values as the earlier ones when there is no difference between the desired and contracted spreads, slightly different otherwise.

In the global FRN market it is customary to quote prices by means of discount margin, i.e. the desired spread over the reference rate.

15. Duration of a floating rate bond.

The duration of a floating rate bond with no contracted premium over the reference rate is the remaining period till the next refixation. While this is a good enough definition for most practical purposes when then the coupon incorporates a margin over the reference rate, a more rigorous approach leads to a slightly modification.

A floating rate bond carrying a margin, typically premium, over the reference rate, can be looked at as a portfolio of two bonds:

- a floating rate bond with coupon equal to the reference rate; and
- a fixed rate bond with coupon equal to the contracted premium over the benchmark rate.

The duration of the first bond can be calculated as desired in the previous paragraph; that of the second in the usual way for fixed income securities. The duration of the floating rate bond with coupon set at a premium to the reference rate can then be calculated as the duration of the two bond portfolios.

This methodology gives a somewhat higher duration than calculated ignoring the premium over the reference rate.